Period 1, 8/26/2024

$$\frac{3\cdot 1}{3\cdot(3+x)} - \frac{1(3+x)}{3(3+x)} = \frac{3}{3(3+x)} - \frac{3+x}{3(3+x)}$$

$$\frac{\cancel{3}-\cancel{3}-x}{3(3+x)}$$

e)

$$\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x}$$

$$\lim_{x\to 0}\frac{\frac{-x}{3(3+x)}}{\frac{x}{1}} = \lim_{x\to 0}\frac{-\cancel{x}}{3(3+x)}\cdot\frac{1}{\cancel{x}} = \lim_{x\to 0}\frac{-1}{3(3+x)} = \frac{-1}{3\cdot 3} = \frac{-1}{9}$$

$$\lim_{x\to 0}\frac{\cos x-1}{x} \le \lim_{x\to 0} h(x) \le \lim_{x\to 0}(e^x-1) \Rightarrow 0 \le \lim_{x\to 0} h(x) \le 0$$

2. Given $\frac{\cos x - 1}{x} \le h(x) \le e^x - 1$, find $\lim_{x\to 0} h(x)$.

Squeeze Theorem

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$

$$\lim_{x\to 0}(e^x - 1) = 1 - 1 = 0$$

$$\lim_{x\to 0} h(x) = 0$$

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c)

$$\lim_{x\to 1}\frac{x^2-3x+2}{x^2-7x+6} = \frac{1-3+2}{1-7+6} = \frac{0}{0} = \emptyset$$

$$x^2-3x+2 = (x-2)(x-1)$$

$$x^2-7x+6 = (x-6)(x-1)$$

$$\lim_{x\to 1}\frac{(x-2)\cancel{(x-1)}}{(x-6)\cancel{(x-1)}} = \lim_{x\to 1}\frac{x-2}{x-6} = \frac{1-2}{1-6} = \frac{-1}{-5} = \frac{1}{5}$$

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d)

$$\lim_{x\to 3}\frac{(\sqrt{x+6}-3)}{(x-3)} = \frac{\sqrt{3+6}-3}{3-3} = \frac{\sqrt{9}-3}{3-3} = \frac{3-3}{3-3} = \frac{0}{0} = \emptyset$$

$$\frac{(\sqrt{x+6}-3)}{(x-3)}\cdot\frac{(\sqrt{x+6}+3)}{(\sqrt{x+6}+3)} = \lim_{x\to 3}\frac{x+6+\cancel{3\sqrt{x+6}}-\cancel{3\sqrt{x+6}}-9}{(x-3)(\sqrt{x+6}+3)}$$

$\Downarrow$

$$\lim_{x\to 3} \frac{x+6-9}{(x-3)(\sqrt{x+6}+3)} = \lim_{x\to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+6}+3)} = \lim_{x\to 3} \frac{1}{\sqrt{x+6}+3}$$

$$\frac{1}{\sqrt{3+6}+3}$$

$$\frac{1}{\sqrt{9}+3}$$

$$\frac{1}{3+3} = \frac{1}{6}$$

e) $\displaystyle \lim_{x\to 0} \frac{1-\cos 2x}{4x} = \lim_{x\to 0} \frac{1-\cos 2x}{2\cdot 2x} = \lim_{x\to 0} \frac{1}{2}\cdot \cancel{\frac{1-\cos 2x}{2x}}$

0

$$\frac{1}{2}\cdot 0 = 0$$

d) $\displaystyle \lim_{x\to 0} \frac{x+\sin x}{x} = \lim_{x\to 0} \frac{x}{x} + \frac{\sin x}{x} = 1+1 = 2$

$$\lim_{x\to 0} \frac{8x}{\sin 5x} = \lim_{x\to 0} \frac{5\cdot 8x}{5\cdot \sin 5x} = \lim_{x\to 0} \frac{8\cdot \cancel{5x}}{5\cdot \cancel{\sin 5x}} = \frac{8}{5}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0 \qquad \lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

$a = 2x$

$\displaystyle \lim_{x\to 0} 2x = 0$

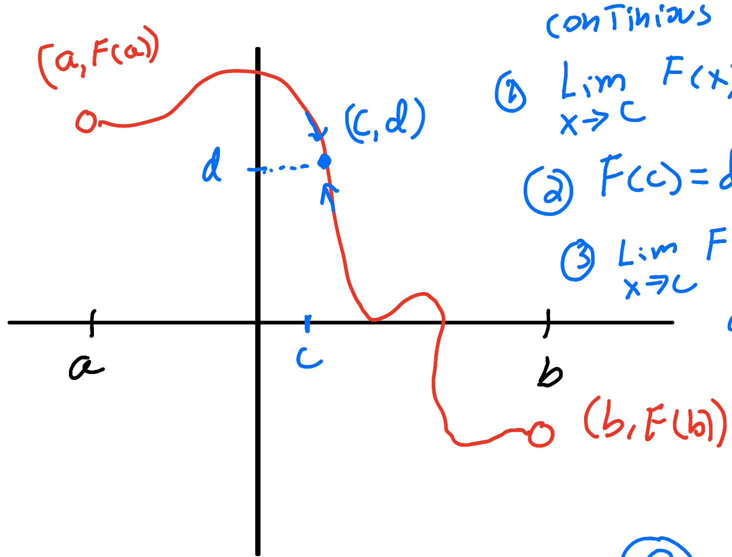
$\displaystyle \lim_{x\to 0} a = 0$

e) $\displaystyle \lim_{x\to 0} \frac{1-\cos 2x}{4x} = \frac{1-\cos a}{2\,a} = \frac{1}{2}\cdot 0 = 0$

$$\lim_{x\to 0} \frac{1-\cos 2x}{2\cdot 2x} = \lim_{x\to 0} \frac{1}{2}\cdot \boxed{\frac{(1-\cos 2x)}{2x}} = \frac{1}{2}\cdot 0$$

0

$F(c)=d$



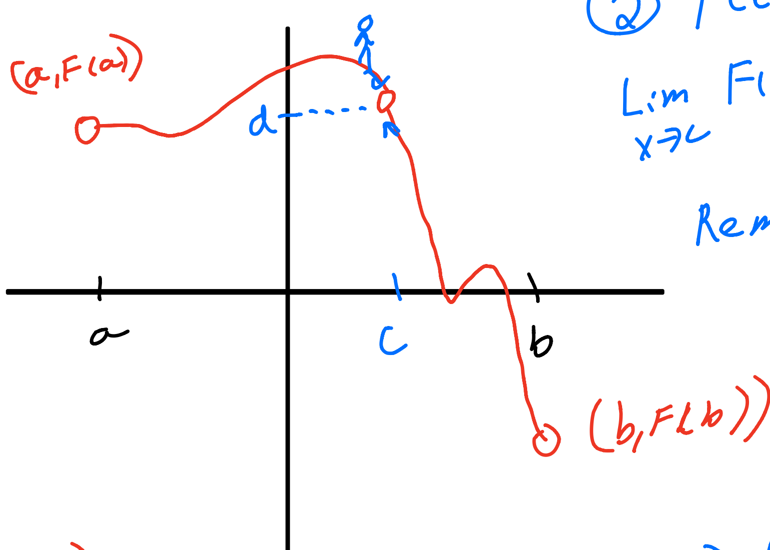
continious at $x=c$

① $\lim_{x\to c} F(x)=d$

② $F(c)=d$

③ $\lim_{x\to c} F(x)=F(c)$

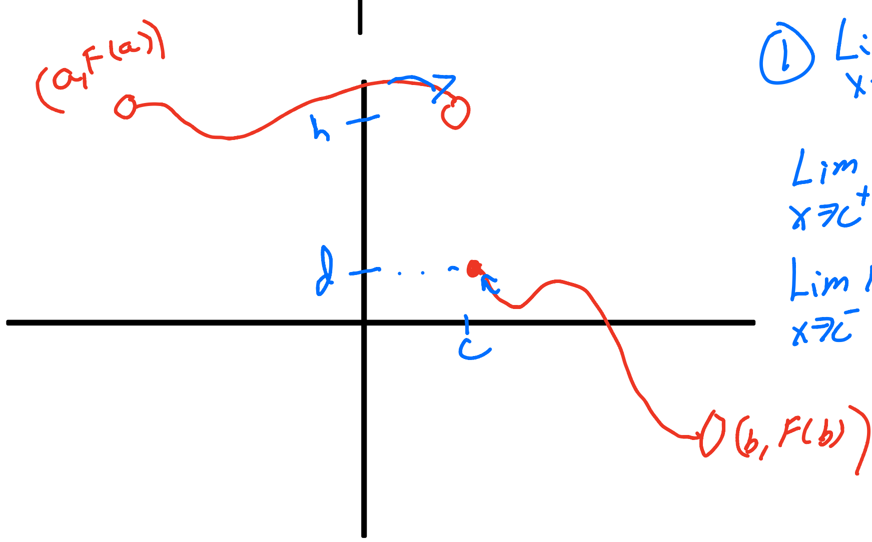
$d=d$



② $F(c)=\emptyset$

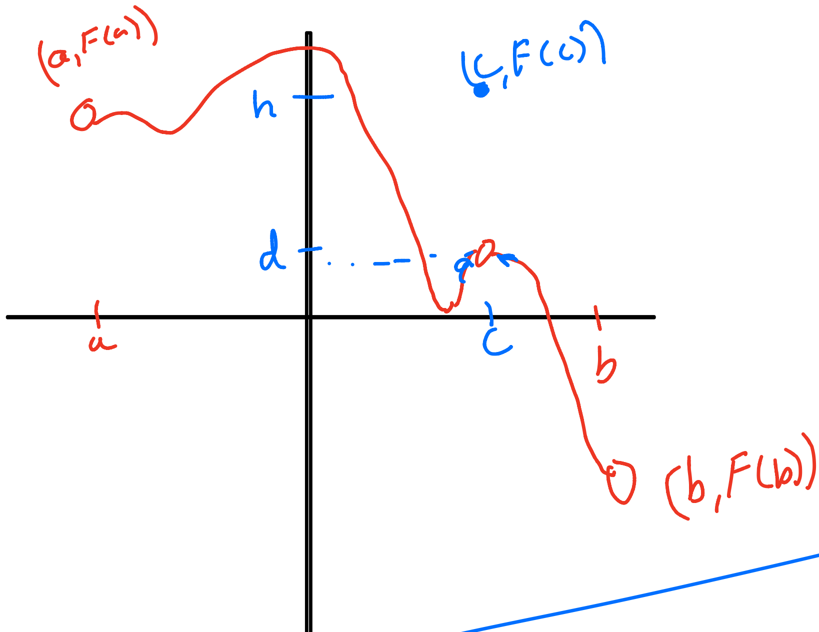
$\lim_{x\to c} F(x)=d$

Removable



① $\lim_{x\to c} F(x)=\emptyset$

$\left.\begin{array}{l}\lim_{x\to c^+} F(x)=d \\ \lim_{x\to c^-} F(x)=h\end{array}\right\}$ Not The Same



$\lim_{x \to c} F(x) = d$

$F(c) = h$

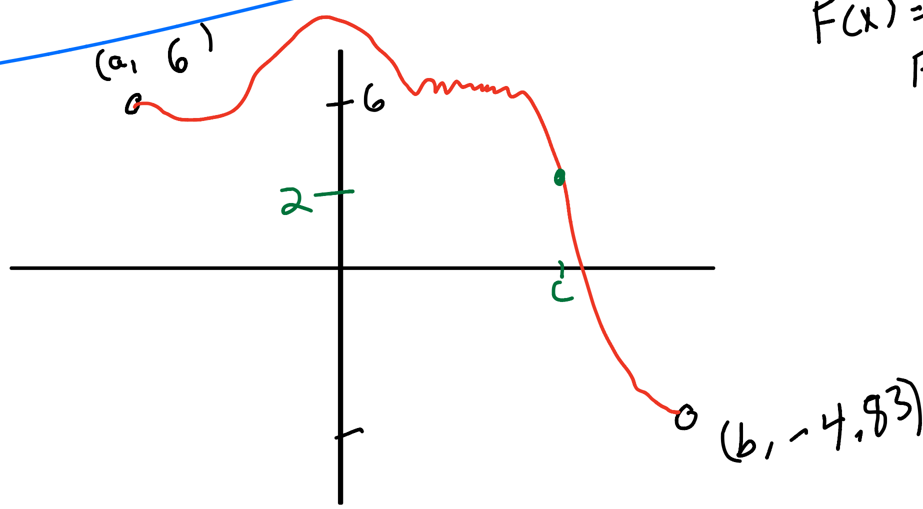
$\lim_{x \to c} F(x) \neq F(c)$

$d \neq h$

Removable

IVT

$F(x)$ = continuous from a to b



Pick any number between 6 and -4.83.

2

The is a number between a and b such that $F(c)=2$

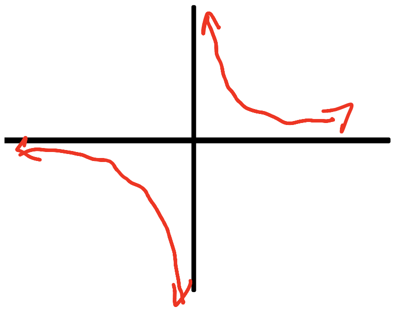
$a<c<b$

Example 2

Discuss the continuity of each function.

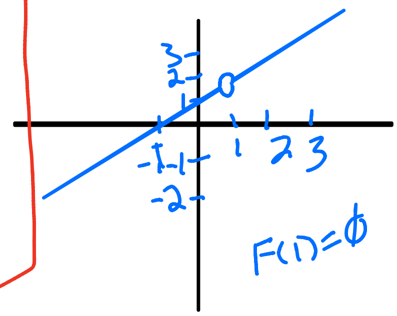
$\lim\limits_{x\to 0} F(x) = \emptyset$ $\quad F(0) = \emptyset$

$$f(x) = \frac{1}{x}$$



$x \neq 1$

$$g(x) = \frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

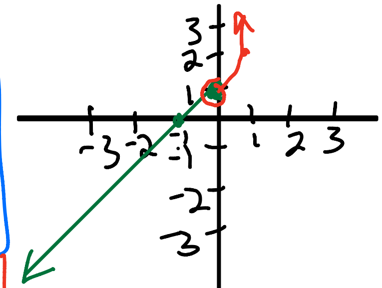


$F(1) = \emptyset$

$h(0) = 0 + 1 = 1$

$$h(x) = \begin{cases} \underline{x+1}, & x \le 0 \\ \underline{x^2+1}, & x > 0 \end{cases}$$

Continuous



$\lim\limits_{x\to 0} h(x) = 1$

$h(0) = 1$

$\lim\limits_{x\to 0} h(x) = h(0)$

$1 = 1$

a) $$g(x) = \begin{cases} x^2 + 7, & x \ge 1 \\ x + a, & x < 1 \end{cases}$$

Continuous

$\lim\limits_{x\to 1^+} g(x) = \lim\limits_{x\to 1^+} x^2+7 = 8$

$g(1) = 1^2 + 7 = 8$ True

$\lim\limits_{x\to 1} g(x) = 8$

$\lim\limits_{x\to 1^-} g(x) = \lim\limits_{x\to 1^-} x+a = 8$

$1 + a = 8$

$a = 7$

b) $h(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1}, & x \neq 1 \\ a, & x = 1 \end{cases}$

$x \neq 1$

$\dfrac{(x^2-1)(x^2+1)}{x-1} = \dfrac{(x-1)(x+1)(x^2+1)}{x-1}$

$\lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{(x-1)}} = (1+1)(1^2+1) = 2 \cdot 2 = 4$

$F(1) = 4$

$F(1) = a$

$\lim\limits_{x \to 1} \dfrac{x^4-1}{x-1} = F(1) \longrightarrow 4 = a$

Example 6

t	5	6	7	8	9
$P(t)$	15	11	10	8	6

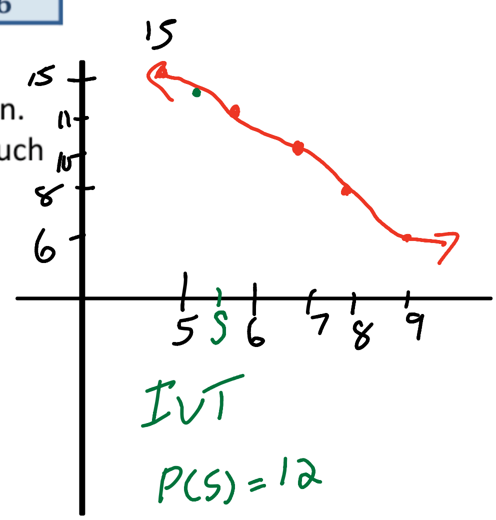
12

$15 < 12 < 11$

Selected values of the function $P(t)$ are given in the table above. $P(t)$ is a continuous decreasing function. Explain why there must be a value s for $5 < s < 9$ such that $P(s) = 12$.

$P(5) = 15$

$P(9) = 6$



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$P(s) = 12$

$5 < s < 6$

TIP

1) You DO NOT know what is happening between 5 and 6, or ANY two t values.
2) Plot points listed

Example 5

Explain why for the function $h(x) = x^4 + 5x^2 - 7$ there **exist** a c in [1,2] such that $h(c) = 20$.

h(x) is continious

$-1 < 20 < 29$

$h(1) < h(c) < h(2)$

$h(1) = 1^4 + 5(1)^2 - 7 = -1$

$1 < c < 2$

$h(2) = 2^4 + 5(2)^2 - 7 = 16 + 20 - 7 = 29$

IVT

AP® Calculus AB
2007 Free-Response Questions

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

F(x) and g(x) are continious

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

h(x) is continious

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

$h(1) = F(g(1)) - 6 = F(2) - 6 = 9 - 6 = 3$

$h(3) = F(g(3)) = F(4) - 6 = -1 - 6 = -7$

$-7 < -5 < 3$

$h(3) < h(r) < h(1)$

$1 < r < 3$

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

   (a) Show that f is continuous at $x = 0$.